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Hypothesis**W. V. Quine and J. S. Ullian**

Some philosophers once held that whatever was true could in principle be proved from self-evident beginnings by self-evident steps. The trait of absolute demonstrability, which we attributed to the truths of logic in a narrow sense and to relatively little else, was believed by those philosophers to pervade all truth. They thought that but for our intellectual limitations we could find proofs for any truths, and so, in particular, predict the future to any desired extent. These philosophers were the rationalists. Other philosophers, a little less sanguine, had it that whatever was true could be proved by self-evident steps from two-fold beginnings: self-evident truths and observations. Philosophers of both schools, the rationalists and the somewhat less sanguine ones as well, strained toward their ideals by construing self-evidence every bit as broadly as they in conscience might, or somewhat more so.

Actually even the truths of elementary number theory are presumably not in general derivable, we noted, by self-evident steps from self-evident truths. We owe this insight to Godel's theorem, which was not known to the old-time philosophers.

What then of the truths of nature? Might these be derivable still by self-evident steps from self-evident truths

together with observations? Surely not. Take the humblest generalization from observation: that giraffes are mute, that sea water tastes of salt. We infer these from our observations of giraffes and sea water because we expect instinctively that what is true of all observed samples is true of the rest. The principle involved here, far from being self-evident, does not always lead to true generalizations. It worked for the giraffes and the sea water, but it would have let us down if we had inferred from a hundred observations of swans that all swans are white.

Such generalizations already exceed what can be proved from observations and self-evident truths by self-evident steps. Yet such generalizations are still only a small part of natural science. Theories of molecules and atoms are not related to any observations in the direct way in which the generalizations about giraffes and sea water are related to observations of mute giraffes and salty sea water.

It is now recognized that deduction from self-evident truths and observation is not the sole avenue to truth nor even to reasonable belief. A dominant further factor, in solid science as in daily life, is *hypothesis*. In a word, hypothesis is guesswork; but it can be enlightened guesswork.

It is the part of scientific rigor to recognize hypothesis as hypothesis and then to make the most of it. Having accepted the fact that our observations and our self-evident truths do not together suffice to predict the future, we frame hypotheses to make up the shortage.

Calling a belief a hypothesis says nothing as to what the belief is about, how firmly it is held, or how well founded it is. Calling it a hypothesis suggests rather what sort of reason we have for adopting or entertaining it. People adopt or

entertain a hypothesis because it would explain, if it were true, some things that they already believe. Its evidence is seen in its consequences. . . .

Hypothesis, when successful, is a two-way street, extending back to explain the past and forward to predict the future. What we try to do in framing hypotheses is to explain some otherwise unexplained happenings by inventing a plausible story, a plausible description or history of relevant portions of the world. What counts in favor of a hypothesis is a question not to be lightly answered. We may note five virtues that a hypothesis may enjoy in varying degrees.

Virtue I is *conservatism*. In order to explain the happenings that we are inventing it to explain, the hypothesis may have to conflict with some of our previous beliefs; but the fewer the better. Acceptance of a hypothesis is of course like acceptance of any belief in that it demands rejection of whatever conflicts with it. The less rejection of prior beliefs required, the more plausible the hypothesis—other things being equal.

Often some hypothesis is available that conflicts with no prior beliefs. Thus we may attribute a click at the door to arrival of mail through the slot. Conservatism usually prevails in such a case; one is not apt to be tempted by a hypothesis that upsets prior beliefs when there is no need to resort to one. When the virtue of conservatism deserves notice, rather, is when something happens that cannot evidently be reconciled with our prior beliefs.

There could be such a case when our friend the amateur magician tells us what card we have drawn. How did he do it? Perhaps by luck, one chance in fifty-two; but this conflicts with our reasonable belief, if all unstated, that he would not

have volunteered a performance that depended on that kind of luck. Perhaps the cards were marked; but this conflicts with our belief that he had had no access to them, they being ours. Perhaps he peeked or pushed, with help of a sleight-of-hand; but this conflicts with our belief in our perceptiveness. Perhaps he resorted to telepathy or clairvoyance; but this would wreak havoc with our whole web of belief. The counsel of conservatism is the sleight-of-hand.

Conservatism is rather effortless on the whole, having inertia in its favor. But it is sound strategy too, since at each step it sacrifices as little as possible of the evidential support, whatever that may have been, that our overall system of beliefs has hitherto been enjoying. The truth may indeed be radically remote from our present system of beliefs, so that we may need a long series of conservative steps to attain what might have been attained in one rash leap. The longer the leap, however, the more serious an angular error in the direction. For a leap in the dark the likelihood of a happy landing is severely limited. Conservatism holds out the advantages of limited liability and a maximum of live options for each next move.

Virtue II, closely akin to conservatism, is *modesty*. One hypothesis is more modest than another if it is weaker in a logical sense: if it is implied by the other, without implying it. A hypothesis *A* is more modest than *A* and *B* as a joint hypothesis. Also, one hypothesis is more modest than another if it is more humdrum: that is, if the events that it assumes to have happened are of a more usual and familiar sort, hence more to be expected. Thus suppose a man rings our telephone and ends by apologizing for dialing the wrong number. We will guess that he slipped, rather than that he was a burglar

checking to see if anyone was home. It is the more modest of the two hypotheses, butterfingers being rife. We could be wrong, for crime is rife too. But still the butterfingers hypothesis scores better on modesty than the burglar hypothesis, butterfingers being rifer.

We habitually practice modesty, all unawares, when we identify recurrent objects. Unhesitatingly we recognize our car off there where we parked it, though it may have been towed away and another car of the same model may have happened to pull in at that spot. Ours is the more modest hypothesis, because staying put is a more usual and familiar phenomenon than the alternative combination.

It tends to be the counsel of modesty that the lazy world is the likely world. We are to assume as little activity as will suffice to account for appearances. This is not all there is to modesty. It does not apply to the preferred hypothesis in the telephone example, since Mr. Butterfingers is not assumed to be a less active man than one who might have plotted burglary. Modesty figured there merely in keeping the assumptions down, rather than in actually assuming inactivity. In the example of the parked car, however, the modest hypothesis does expressly assume there to be less activity than otherwise. This is a policy that guides science as well as common sense. It is even erected into an explicit principle of mechanics under the name of the law of least action.

Between modesty and conservatism there is no call to draw a sharp line. But by Virtue I we meant conservatism only in a literal sense—conservation of past beliefs. Thus there remain grades of modesty still to choose among even when Virtue I—compatibility with previous beliefs—is achieved to perfection; for both a slight hypothesis and an

extravagant one might be compatible with all previous beliefs.

Modesty grades off in turn into Virtue III, *simplicity*. Where simplicity considerations become especially vivid is in drawing curves through plotted points on a graph. Consider the familiar practice of plotting measurements. Distance up the page represents altitude above sea level, for instance, and distance across represents the temperature of boiling water. We plot our measurements on the graph, one dot for each pair. However many points we plot, there remain infinitely many curves that may be drawn through them. Whatever curve we draw represents our generalization from the data, our prediction of what boiling temperatures would be found at altitudes as yet untested. And the curve we will choose to draw is the simplest curve that passes through or reasonably close to all the plotted points.

There is a premium on simplicity in any hypothesis, but the highest premium is on simplicity in the giant joint hypothesis that is science, or the particular science, as a whole. We cheerfully sacrifice simplicity of a part for greater simplicity of the whole when we see a way of doing so. Thus consider gravity. Heavy objects tend downward: here is an exceedingly simple hypothesis, or even a mere definition. However, we complicate matters by accepting rather the hypothesis that the heavy objects around us are slightly attracted also by one another, and by the neighboring mountains, and by the moon, and that all these competing forces detract slightly from the downward one. Newton propounded this more complicated hypothesis even though, aside from tidal effects of the moon, he had no means of detecting the competing forces; for it meant a great gain in the

simplicity of physics as a whole. His hypothesis of universal gravitation, which has each body attracting each in proportion to mass and inversely as the square of the distance, was what enabled him to make a single neat system of celestial and terrestrial mechanics.

A modest hypothesis that was long supported both by theoretical considerations and by observation is that the trajectory of a projectile is a parabola. A contrary hypothesis is that the trajectory deviates imperceptibly from a parabola, constituting rather one end of an ellipse whose other end extends beyond the center of the earth. This hypothesis is less modest, but again it conduces to a higher simplicity: Newton's laws of motion and, again, of gravitation. The trajectories are brought into harmony with Kepler's law of the elliptical orbits of the planets.

Another famous triumph of this kind was achieved by Count Rumford and later physicists when they showed how the relation of gas pressure to temperature could be accounted for by the impact of oscillating particles, for in this way they reduced the theory of gases to the general laws of motion. Such was the kinetic theory of gases. In order to achieve it they had to add the hypothesis, by no means a modest one, that gas consists of oscillating particles or molecules; but the addition is made up for, and much more, by the gain in simplicity accruing to physics as a whole.

What is simplicity? For curves we can make good sense of it in geometrical terms. A simple curve is continuous, and among continuous curves the simplest are perhaps those whose curvature changes most gradually from point to point. When scientific laws are expressed in equations, as they so often are, we can make good sense of simplicity in terms of

what mathematicians call the degree of an equation, or the order of a differential equation. This line was taken by Sir Harold Jeffreys. The lower the degree, the lower the order, and the fewer the terms, the simpler the equation. Such simplicity ratings of equations agree with the simplicity ratings of curves when the equations are plotted as in analytical geometry.

Simplicity is hard to define when we turn away from curves and equations. Sometimes in such cases it is not to be distinguished from modesty. Commonly a hypothesis *A* will count as simpler than *A* and *B* together; thus far simplicity and modesty coincide. On the other hand the simplicity gained by Newton's hypothesis of universal gravitation was not modesty, in the sense that we have assigned to that term; for the hypothesis was not logically implied by its predecessors, nor was it more humdrum in respect of the events that it assumed. Newton's hypothesis was simpler than its predecessors in that it covered in a brief unified story what had previously been covered only by two unrelated accounts. Similar remarks apply to the kinetic theory of gases.

In the notion of simplicity there is a nagging subjectivity. What makes for a brief unified story depends on the structure of our language, after all, and on our available vocabulary, which need not reflect the structure of nature. This subjectivity of simplicity is puzzling, if simplicity in hypotheses is to make for plausibility. Why should the subjectively simpler of two hypotheses stand a better chance of predicting objective events? Why should we expect nature to submit to our subjective standard of simplicity?

That would be too much to expect. Physicists and others are continually finding that they have to complicate their

theories to accommodate new data. At each stage, however, when choosing a hypothesis subject to subsequent correction, it is still best to choose the simplest that is not yet excluded. This strategy recommends itself on much the same grounds as the strategies of conservatism and modesty. The longer the leap, we reflected, the more and wilder ways of going wrong. But likewise, the more complex the hypothesis, the more and wilder ways of going wrong; for how can we tell which complexities to adopt? Simplicity, like conservatism and modesty, limits liability. Conservatism can be good strategy even though one's present theory be ever so far from the truth, and simplicity can be good strategy even though the world be ever so complicated. Our steps toward the complicated truth can usually be laid out most dependably if the simplest hypothesis that is still tenable is chosen at each step. It has even been argued that this policy will lead us at least asymptotically toward a theory that is true.

There is more, however, to be said for simplicity: the simplest hypothesis often just is the likeliest, apparently, quite apart from questions of cagy strategy. Why should this be? There is a partial explanation in our ways of keeping score on predictions. The predictions based on the simpler hypotheses tend to be scored more leniently. Thus consider curves, where simplicity comparisons are so clear. If a curve is kinky and complex, and if some measurement predicted from the curve turns out to miss the mark by a distance as sizable as some of the kinks of the curve itself, we will count the prediction a failure. We will feel that so kinky a curve, if correct, would have had a kink to catch this wayward point. On the other hand, a miss of the same

magnitude might be excused if the curve were smooth and simple. It might be excused as due to inaccuracy of measurement or to some unexplained local interference. This cynical doctrine of selective leniency is very plausible in the case of the curves. And we may reasonably expect a somewhat similar but less easily pictured selectivity to be at work in the interest of the simple hypotheses where curves are not concerned.

Considering how subjective our standards of simplicity are, we wondered why we should expect nature to submit to them. Our first answer was that we need not expect it; the strategy of favoring the simple at each step is good anyway. Now we have noted further that some of nature's seeming simplicity is an effect of our bookkeeping. Are we to conclude that the favoring of simplicity is entirely our doing, and that nature is neutral in the matter? Not quite. Darwin's theory of natural selection offers a causal connection between subjective simplicity and objective truth in the following way. Innate subjective standards of simplicity that make people prefer some hypotheses to others will have survival value insofar as they favor successful prediction. Those who predict best are likeliest to survive and reproduce their kind, in a state of nature anyway, and so their innate standards of simplicity are handed down. Such standards will also change in the light of experience, becoming still better adapted to the growing body of science in the course of the individual's lifetime. (But these improvements do not get handed down genetically.)

Virtue IV is *generality*. The wider the range of application of a hypothesis, the more general it is. When we find electricity conducted by a piece of copper wire, we leap to the hypothesis that all copper, not just long thin copper,

conducts electricity.

The plausibility of a hypothesis depends largely on how compatible the hypothesis is with our being observers placed at random in the world. Funny coincidences often occur, but they are not the stuff that plausible hypotheses are made of. The more general the hypothesis is by which we account for our present observation, the less of a coincidence it is that our present observation should fall under it. Hence, in part, the power of Virtue IV to confer plausibility.

The possibility of testing a hypothesis by repeatable experiment presupposes that the hypothesis has at least some share of Virtue IV. For in a repetition of an experiment the test situation can never be exactly what it was for the earlier run of the experiment; and so, if both runs are to be relevant to the hypothesis, the hypothesis must be at least general enough to apply to both test situations.¹ One would of course like to have it much more general still.

Virtues I, II, and III made for plausibility. So does Virtue IV to some degree, we see, but that is not its main claim; indeed generality conflicts with modesty. But generality is desirable in that it makes a hypothesis interesting and important if true.

We lately noted a celebrated example of generality in Newton's hypothesis of universal gravitation, and another in the kinetic theory of gases. It is no accident that the same illustrations should serve for both simplicity and generality. Generality without simplicity is cold comfort. Thus take celestial mechanics with its elliptical orbits, and take also terrestrial mechanics with its parabolic trajectories, just take them in tandem as a bipartite theory of motion. If the two together cover everything covered by Newton's unified laws

of motion, then generality is no ground for preferring Newton's theory to the two taken together. But Virtue II, simplicity, is. When a way is seen of gaining great generality with little loss of simplicity, or great simplicity with no loss of generality, the conservatism and modesty give way to scientific revolution.

The aftermath of the famous Michelson-Morley experiment of 1887 is a case in point. The purpose of this delicate and ingenious experiment was to measure the speed with which the earth travels through the ether. For two centuries, from Newton onward, it had been a well-entrenched tenet that something called the ether pervaded all of what we think of as empty space. The great physicist Lorentz (1853–1928) had hypothesized that the ether itself was stationary. What the experiment revealed was that the method that was expected to enable measurement of the earth's speed through the ether was totally inadequate to that task. Supplementary hypotheses multiplied in an attempt to explain the failure without seriously disrupting the accepted physics. Lorentz, in an effort to save the hypothesis of stationary ether, shifted to a new and more complicated set of formulas in his mathematical physics. Einstein soon cut through all this, propounding what is called the special theory of relativity.

This was a simplification of physical theory. Not that Einstein's theory is as simple as Newton's had been; but Newton's physics had been shown untenable by the Michelson-Morley experiment. The point is that Einstein's theory is simpler than Newton's as corrected and supplemented and complicated by Lorentz and others. It was a glorious case of gaining simplicity at the sacrifice of conservatism; for the time-honored ether went by the board, and far older and more

fundamental tenets went by the board too. Drastic changes were made in our conception of the very structure of space and time. . . .

Yet let the glory not blind us to Virtue I. When our estrangement from the past is excessive, the imagination boggles; genius is needed to devise the new theory, and high talent is needed to find one's way about in it. Even Einstein's revolution, moreover, had its conservative strain; Virtue I was not wholly sacrificed. The old physics of Newton's classical mechanics is, in a way, preserved after all. For the situations in which the old and the new theories would predict contrary observations are situations that we are not apt to encounter without sophisticated experiment—because of their dependence on exorbitant velocities or exorbitant distances. This is why classical mechanics held the field so long. Whenever, even having switched to Einstein's relativity theory, we dismiss those exorbitant velocities and distances for the purpose of some practical problem, promptly the discrepancy between Einstein's theory and Newton's becomes too small to matter. Looked at from this angle, Einstein's theory takes on the aspect not of a simplification but a generalization. We might say that the sphere of applicability of Newtonian mechanics in its original simplicity was shown, by the Michelson-Morley experiment and related results, to be less than universal; and then Einstein's theory comes as a generalization, presumed to hold universally. Within its newly limited sphere, Newtonian mechanics retains its old utility. What is more, the evidence of past centuries for Newtonian mechanics even carries over, within these limits, as evidence for Einstein's physics; for, as far as it goes, it fits both.

What is thus illustrated by Einstein's relativity is more

modestly exemplified elsewhere, and generally aspired to: the retention, in some sense, of old theories in new ones. If the new theory can be so fashioned as to diverge from the old only in ways that are undetectable in most ordinary circumstances, then it inherits the evidence of the old theory rather than having to overcome it. Such is the force of conservatism even in the context of revolution.

Virtues I through IV may be further illustrated by considering Neptune. That Neptune is among the planets is readily checked by anyone with reference material; indeed it passes as common knowledge, and there is for most of us no need to check it. But only through extensive application of optics and geometry was it possible to determine, in the first instance, that the body we call Neptune exists, and that it revolves around the sun. This required not only much accumulated science and mathematics, but also powerful telescopes and cooperation among scientists.

In fact it happens that Neptune's existence and planethood were strongly suspected even before that planet was observed. Physical theory made possible the calculation of what the orbit of the planet Uranus should be, but Uranus' path differed measurably from its calculated course. Now the theory on which the calculations were based was, like all theories, open to revision or refutation. But here conservatism operates: one is loath to revise extensively a well-established set of beliefs, especially a set so deeply entrenched as a basic portion of physics. And one is even more loath to abandon as spurious immense numbers of observation reports made by serious scientists. Given that Uranus had been observed to be as much as two minutes of arc from its calculated position, what was sought was a discovery that would render this

deviation explicable within the framework of accepted theory. Then the theory and its generality would be unimpaired, and the new complexity would be minimal.

It would have been possible in principle to speculate that some special characteristic of Uranus exempted that planet from the physical laws that are followed by other planets. If such a hypothesis had been resorted to, Neptune would not have been discovered; not then, at any rate. There was a reason, however, for not resorting to such a hypothesis. It would have been what is called an *ad hoc hypothesis*, and ad hoc hypotheses are bad air; for they are wanting in Virtues III and IV. Ad hoc hypotheses are hypotheses that purport to account for some particular observations by supposing some very special forces to be at work in the particular cases at hand, and not generalizing sufficiently beyond those cases. The vice of an ad hoc hypothesis admits of degrees. The extreme case is where the hypothesis covers only the observations it was invented to account for, so that it is totally useless in prediction. Then also it is insusceptible of confirmation, which would come of our verifying its predictions.

Another example that has something of the implausibility of an ad hoc hypothesis is the water-diviner's belief that a willow wand held above the ground can be attracted by underground water. The force alleged is too special. One feels, most decidedly, the lack of an intelligible mechanism to explain the attraction. And what counts as intelligible mechanism? A hypothesis strikes us as giving an intelligible mechanism when the hypothesis rates well in familiarity, generality, simplicity. We attain the ultimate in intelligibility of mechanism, no doubt, when we see how to explain something in terms of physical impact or the familiar and

general laws of motion.

There is an especially notorious sort of hypothesis which, whether or not properly classified also as ad hoc, shares the traits of insusceptibility of confirmation and uselessness in prediction. This is the sort of hypothesis that seeks to save some other hypothesis from refutation by systematically excusing the failures of its predictions. When the Voice from Beyond is silent despite the incantations of the medium, we may be urged to suppose that "someone in the room is interfering with the communication." In an effort to save the prior hypothesis that certain incantations will summon forth the Voice, the auxiliary hypothesis that untoward thoughts can thwart audible signals is advanced. This auxiliary hypothesis is no wilder than the hypothesis that it was invoked to save, and thus an uncritical person may find the newly wrinkled theory no harder to accept than its predecessor had been. On the other hand the critical observer sees that evidence has ceased altogether to figure. Experimental failure is being milked to fatten up theory.

These reflections bring a fifth virtue to the fore: *refutability*, Virtue V. It seems faint praise of a hypothesis to call it refutable. But the point, we have now seen, is approximately this: some imaginable event, recognizable if it occurs, must suffice to refute the hypothesis. Otherwise the hypothesis predicts nothing, is confirmed by nothing, and confers upon us no earthly good beyond perhaps a mistaken peace of mind.

This is too simple a statement of the matter. Just about any hypothesis, after all, can be held unrefuted no matter what, by making enough adjustments in other beliefs—though sometimes doing so requires madness. We think loosely of a

hypothesis as implying predictions when, strictly speaking, the implying is done by the hypothesis together with a supporting chorus of ill-distinguished background beliefs. It is done by the whole relevant theory taken together.

Properly viewed, therefore, Virtue V is a matter of degree, as are its four predecessors. The degree to which a hypothesis partakes of Virtue V is measured by the cost of retaining the hypothesis in the face of imaginable events. The degree is measured by how dearly we cherish the previous beliefs that would have to be sacrificed to save the hypothesis. The greater the sacrifice, the more refutable the hypothesis.

A prime example of deficiency in respect of Virtue V is astrology. Astrologers can so hedge their predictions that they are devoid of genuine content. We may be told that a person will "tend to be creative" or "tend to be outgoing," where the evasiveness of a verb and the fuzziness of adjectives serve to insulate the claim from repudiation. But even if a prediction should be regarded as a failure, astrological devotees can go on believing that the stars rule our destinies; for there is always some item of information, perhaps as to a planet's location at a long gone time, that may be alleged to have been overlooked. Conflict with other beliefs thus need not arise.

All our contemplating of special virtues of hypotheses will not, we trust, becloud the fact that the heart of the matter is observation. Virtues I through V are guides to the framing of hypotheses that, besides conforming to past observations, may plausibly be expected to conform to future ones. When they fail on the latter score, questions are reopened. Thus it was that the Michelson-Morley experiment

led to modifications, however inelegant, of Newton's physics at the hands of Lorentz. When Einstein came out with a simpler way of accommodating past observations, moreover, his theory was no mere reformulation of the Newton-Lorentz system; it was yet a third theory, different in some of its predicted observations and answerable to them. Its superior simplicity brought plausibility to its distinctive consequences.

Hypotheses were to serve two purposes: to explain the past and predict the future. Roughly and elliptically speaking, the hypothesis serves these purposes by implying the past events that it was supposed to explain, and by implying future ones. More accurately speaking, as we saw, what does the implying is the whole relevant theory taken together, as newly revised by adoption of the hypothesis in question. Moreover, the predictions that are implied are mostly not just simple predictions of future observations or other events; more often they are conditional predictions. The hypothesis will imply that we will make these further observations if we look in such and such a place, or take other feasible steps. If the predictions come out right, we can win bets or gain other practical advantages. Also, when they come out right, we gain confirmatory evidence for our hypotheses. When they come out wrong, we go back and tinker with our hypotheses and try to make them better.

What we called limiting principles in chapter 4 are, when intelligible, best seen as hypotheses—some good, some bad. Similarly, of course, for scientific laws generally. And similarly for laws of geometry, set theory, and other parts of mathematics. All these laws—those of physics and those of mathematics equally—are among the component hypotheses that fit together to constitute our inclusive scientific theory of

the world. The most general hypotheses tend to be the least answerable to any particular observation, since subsidiary hypotheses can commonly be juggled and adjusted to accommodate conflicts; and on this score of aloofness there is no clear boundary between theoretical physics and mathematics. Of course hypotheses in various fields of inquiry may tend to receive their confirmation from different kinds of investigation, but this should in no way conflict with our seeing them all as hypotheses.

We talk of framing hypotheses. Actually we inherit the main ones, growing up as we do in a going culture. The continuity of belief is due to the retention, at each particular time, of most beliefs. In this retentiveness science even at its most progressive is notably conservative. Virtue I looms large. A reasonable person will look upon some of his or her retained beliefs as self-evident, on others as common knowledge though not self-evident, on others as vouched for by authority in varying degree, and on others as hypotheses that have worked all right so far.

But the going culture goes on, and each of us participates in adding and dropping hypotheses. Continuity makes the changes manageable. Disruptions that are at all sizable are the work of scientists, but we all modify the fabric in our small way, as when we conclude on indirect evidence that the schools will be closed and the planes grounded or that an umbrella thought to have been forgotten by one person was really forgotten by another.

NOTE

1. **We are indebted to Nell E. Scroggins for suggesting this point.**